Kalman Filter vs Alternative Modeling Techniques and Applied Investment Strategies

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Overview

- Introduction
- Literature review
- Research question
- Recursive OLS methods
- State-space model
  - Kalman filter
  - Expectation-Maximization
- Data
- Empirical results
- Economic significance
  - Market timing
  - Pairs trading
- Conclusion
- Q & A

Introduction

- Financial time-series often experience parameter instability (i.e., time-variation in coefficients) —such as stock-bond correlations or investors' risk-preferences
- We examine the efficacy of alternative modeling techniques to predict stock market returns modeled with time-varying coefficients
- The degree to which the time evolution of these parameters is captured depends on the estimation procedure
- Our task is to try and identify the modeling technique which best estimates these time-varying parameters as it will allow us to markedly improve our forecasts and the potential profitability of a strategy

Literature Review

- Faff, Hillier & Hillier (2000) and Mergner & Bulla (2005) examine the performance of GARCH models and the Kalman Filter to model time-varying systematic risk and conditional volatility.
- Rytchkov (2007) applies the Kalman Filter approach to the analysis of stock return predictability. He models the time-dependence of expected dividends and expected returns specifying the Kalman Filter as a bivariate AR(1).
- Choudhry and Wu (2008) investigate alternative time-varying parameter modeling techniques to forecast UK returns—found state-space model Kalman Filter outperformed GARCH models.
- Dangl and Halling (2011) find predictive estimation procedures that model time-varying parameters have greater forecast ability when forecasting S&P 500 returns.

Research Question

Can forecasts, in general, be improved by using a state-space model (SSM) instead of regression when coefficients are time-dependent? Will the improvement be sustainable in a realistic trading environment?

Objective

Improve the profitability of our investment strategies with the identification of the superior modeling technique.

Hypotheses

H_0: The Kalman Filter approach does not yield optimal forecasts when compared to alternative approaches.

H_1: The SSM Kalman Filter approach yields more accurate forecasts when coefficients are time-dependent.

Rolling Window OLS (RWOLS) and Expanding Window OLS (EWOLS)

$$y_{t+1} = \boldsymbol{X}_t \boldsymbol{\beta}_t + \varepsilon_{t+1} \qquad \sim N(0, R) \qquad (1)$$

where

$\boldsymbol{\beta}_t = \begin{bmatrix} a_t \\ b_t \end{bmatrix}$ is the unobservable $(2\ x\ 1)$ vector of the time-varying *parameters of interest*.

y_{t+1} , the scalar stock return, *t* represents time.

$\boldsymbol{X}_t$, the independent variable a $(1\ x\ 2)$ row vector.

ε_{t+1} , the error term assumed to be Gaussian white noise and R, is the variance.

Estimation procedures:

Initial estimation window (in-sample) 25% of the data set or 106 months of a 421-month period

Both procedures forecast 1 month ahead using the same size initial in-sample period. RWOLS rolls forward 1 month while dropping the oldest observation, the in-sample window remains the same size. EWOLS expands the estimation window size. With each iteration both models fix the coefficients formed in the estimation window and use those coefficients to re-forecast 1 month ahead.

State-Space Model—the Kalman Filter

State = value of the parameters at a specific point in time

Space = a subset of $\mathbb{R}$ from which the parameters come

State-space model = estimates and updates a sequence of values for the set of parameters

Kalman filter:

$$y_{t+1} = \boldsymbol{X}_t \boldsymbol{\beta}_t + \varepsilon_{t+1} \qquad \sim N(0, R) \qquad \text{Observation equation (2)}$$

$$\boldsymbol{\beta}_{t+1} = \mathbf{A} + \mathbf{B}\boldsymbol{\beta}_t + \boldsymbol{\upsilon}_{t+1} \qquad \sim N(0, \boldsymbol{\Omega}) \qquad \text{State-transition equation (3)}$$

$\boldsymbol{\beta}_t = \begin{bmatrix} a_t \\ b_t \end{bmatrix}$ the **latent 'unobservable' state** a $(2x1)$ vector.

In Eq. (3) $\mathbf{A}$ is a $(2x1)$ vector of constants. The loading $\mathbf{B}$ is restricted to a $(2x2)$ diagonal matrix.

$\boldsymbol{\upsilon}_{t+1}$ is a $(2x1)$ vector and $\boldsymbol{\Omega}$ is the $(2x2)$ state-transition variance/covariance matrix in Eq. (3).

Scalar ε_{t+1} is the observational noise and scalar R is constant variance in Eq. (2).

Kalman Filter Algorithm

- Next state estimate $= EST_{t+1}$
- Current state estimate $= EST_t$
- Observation $= OBS_t$

- Kalman gain $= KG_t$
- Error in the state estimate equation $= E_{EST\,t}$
- Error in the observation equation $= E_{OBS_t}$

1. $$KG_t = \frac{E_{EST\,t}}{E_{EST\,t} + E_{OBS_t}}$$

$$0 < KG < 1$$

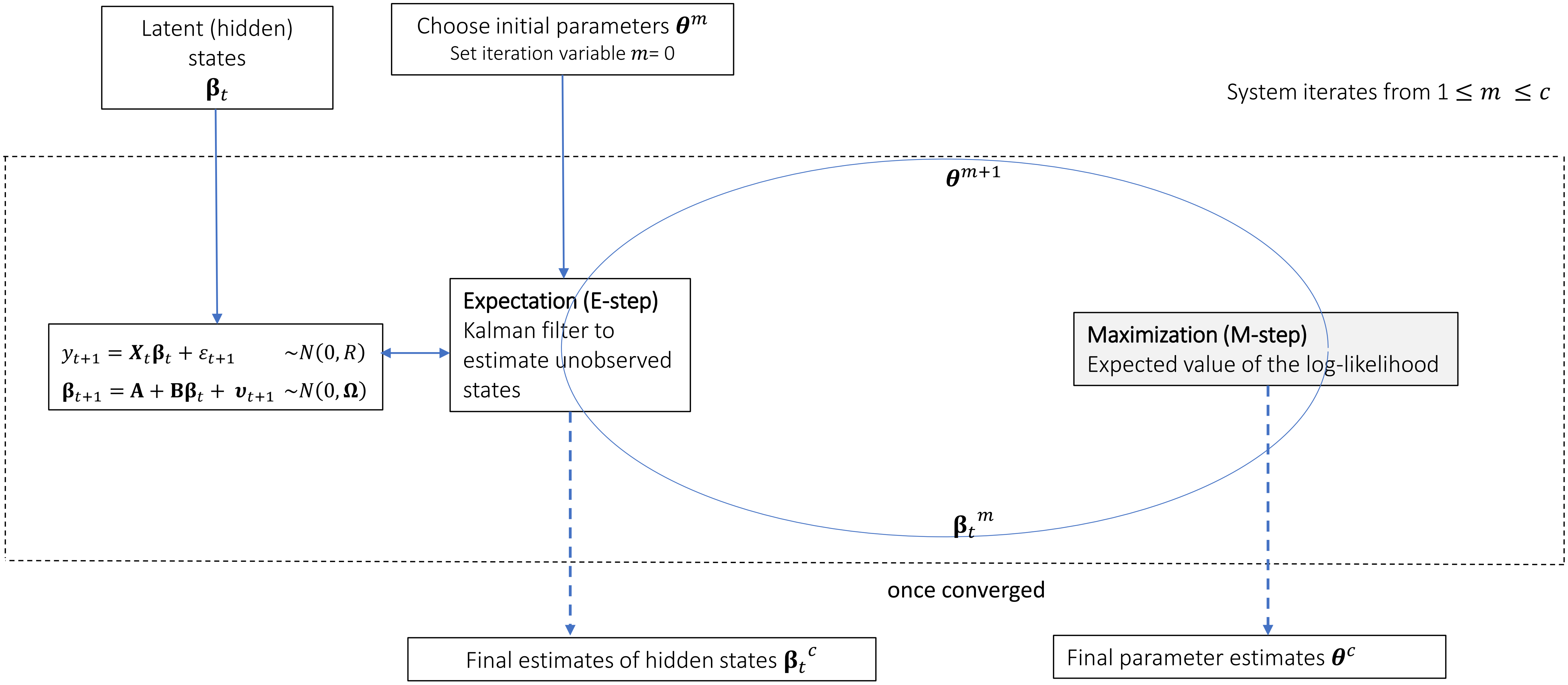
2. $$EST_{t+1} = EST_t + KG_t[OBS_t - EST_t]$$

3. $$E_{EST\,t+1} = [1 - KG_t](E_{EST\,t})$$

1. Calculating the Kalman gain → 2. Calculating the next state estimate → 3. Calculating the new estimate error

Following (van Biezen, 2015)

Expectation-Maximization (EM) Algorithm



Following (Mader et al., 2011) and (Moon, 1995)

Data

Table 1
Variable description.
This table reports monthly data provided by Yahoo Finance, Chicago Board Options Exchange, and Quandl for period March 31st, 1986, to March 31st, 2021. Daily prices for securities VISA and Fidelity National Information Services, Inc. are sourced from ActiveTick for period January 2nd, 2020, to November 16th, 2021.

Symbol	*Variable*
S&P 500	Return on the S&P 500 index
ΔD/P	First differenced dividend-price ratio, lagged one-period.
ΔVIX	First differenced volatility index, lagged one-period.
Lag Return	Lagged return on the S&P 500 index, one-period.
FIS	Security price of Fidelity National Information Services, Inc.
V	Security price of Visa, Inc.

- In-sample March 31st, 1986 – December 31st, 1994, 106 months, 25% of data set
- Out-of-sample January 31st, 1995 – March 31st, 2021, 315 months, remaining 75% of data set
- Securities VISA (V) and Fidelity National Information Services, Inc. (FIS) are used in a separate trading application later in the presentation

Model Results

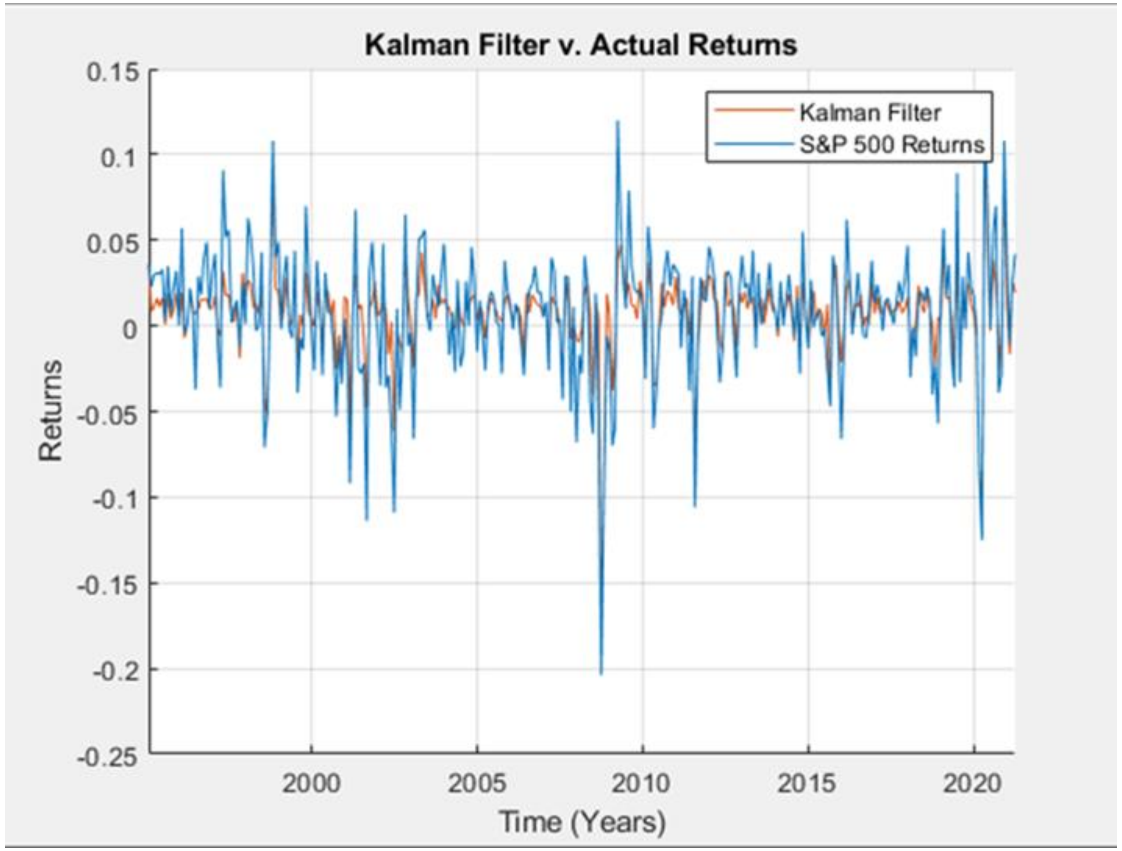
Table 2

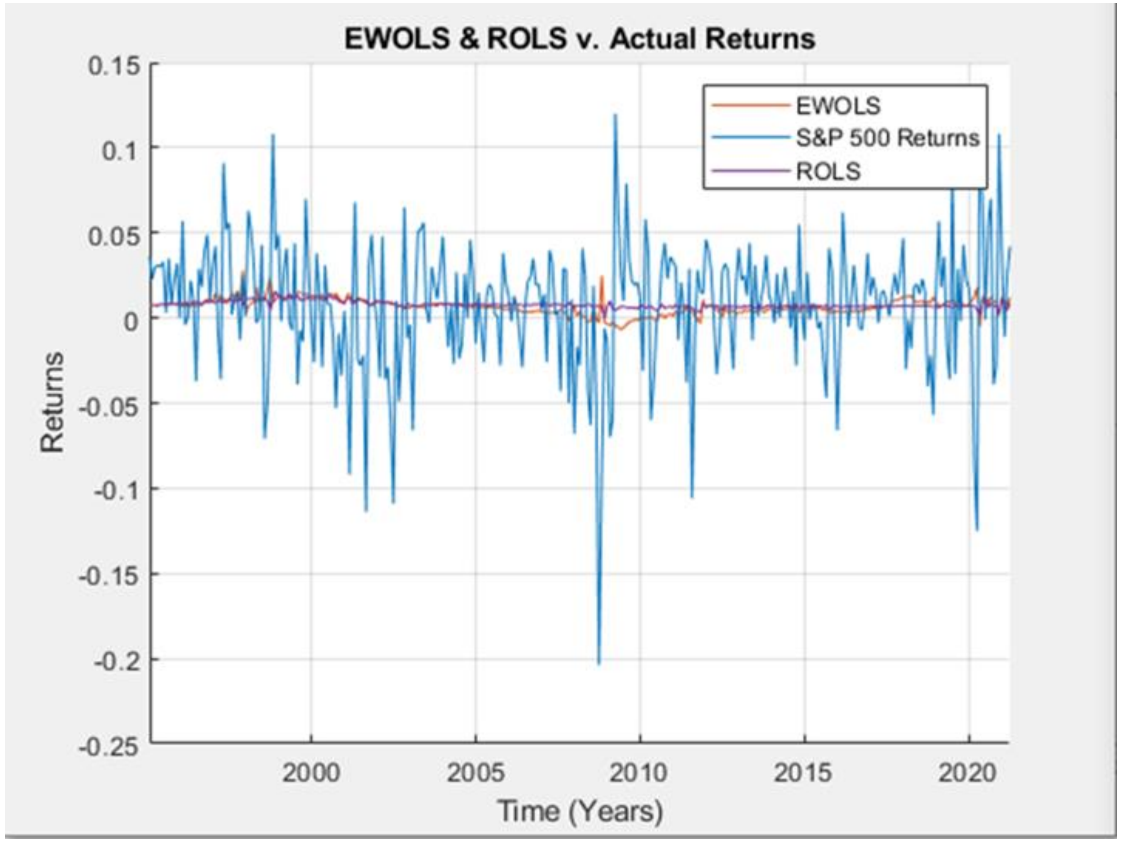
Comparison of OLS beta and point estimates of various beta series.

This table summarizes model performance measures for each technique and independent variable for the out-of-sample period January 31st, 1995, to March 31st, 2021. The mean slope coefficient is reported for each beta series for each modeling technique. Standard errors are reported in parenthesis. *** means that the slope coefficient is significant at the 1% level (**: 5%, *: 10%). In-sample period March 31st, 1986, to December 31st, 1994.

		OLS	*RWOLS*	*EWOLS*	*KF*
Delta VIX	RMSE	3.80%	3.84%	3.82%	3.76%
	MAE	2.76%	2.83%	2.78%	2.73%
	MSE	0.14%	0.15%	0.15%	0.14%
	$OOSR^2$	0.54%	-1.17%	-0.43%	2.62%
	SE	(0.214)	(0.219)	(0.222)	(0.247)
	$\bar{b}$	-0.028	0.004	-1.96	0.058
	t-stat	-12.91***	-0.02	-9.07***	27.40***
	p-value	0.E+00	7.E-02	0.E+00	0.E+00
Lagged S&P	RMSE	3.77%	3.79%	3.77%	3.78%
	MAE	2.71%	2.77%	2.72%	2.72%
	MSE	0.14%	0.14%	0.14%	0.14%
	$OOSR^2$	2.12%	1.50%	2.35%	1.65%
	SE	(0.213)	(0.213)	(0.212)	(0.212)
	$\bar{b}$	0.320	0.202	0.261	0.216
	t-stat	150.60***	96.34***	124.19***	102.33***
	p-value	0.E+00	0.E+00	0.E+00	0.E+00
Dividend/Price	RMSE	3.81%	3.79%	3.82%	3.81%
	MAE	2.76%	2.77%	2.78%	2.76%
	MSE	0.15%	0.14%	0.15%	0.15%
	$OOSR^2$	0.42%	1.50%	-0.41%	0.40%
	SE	(0.214)	(0.213)	(0.215)	(0.215)
	$\bar{b}$	0.000	0.010	-0.003	0.017
	t-stat	-1.E-03	4.74***	0.0137	8.06***
	p-value	9.E-01	3.E-05	2.E-01	0.E+00
	No. Obs.	315	315	315	315

Model Performance





Market Timing Rules

Market timing

Fully invest in the market if the forecasted return is greater than the risk-free rate

Otherwise, if the forecasted market return is less than the risk-free rate, fully invest in the risk-free rate.

Buy and Hold

Buy and hold the market portfolio for the entire period.

Market Timing Results

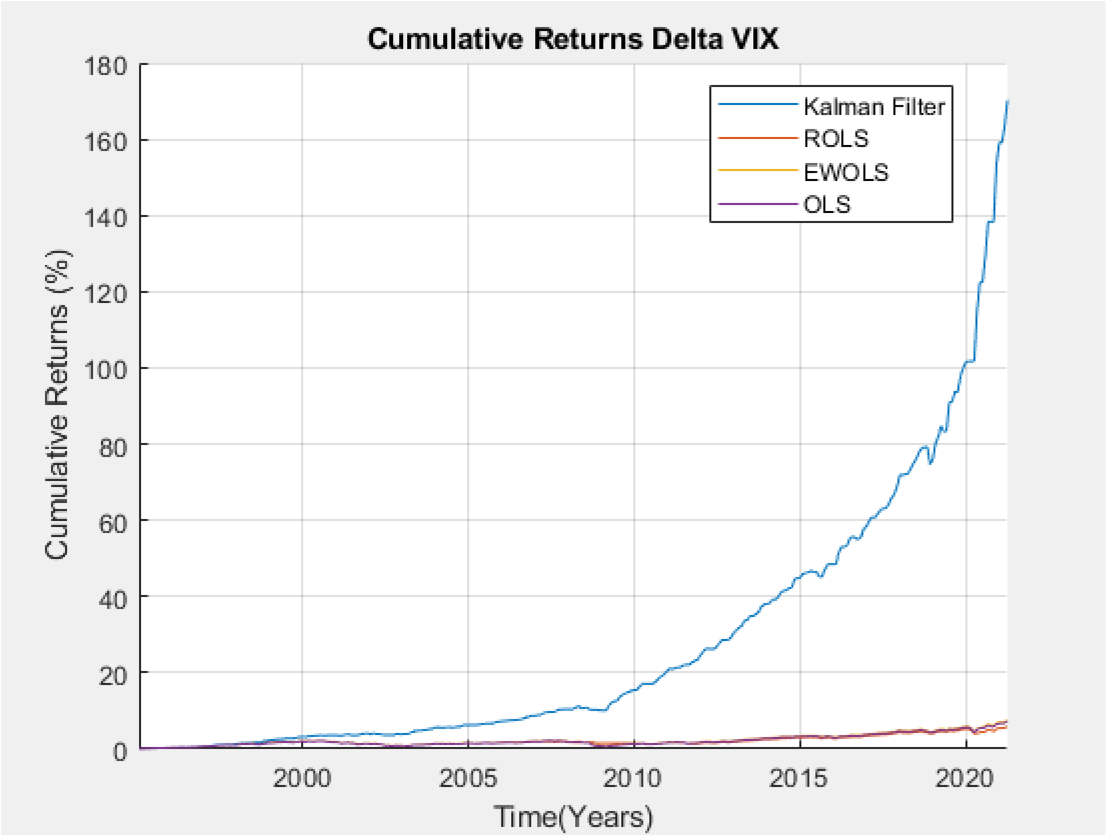
Table 3

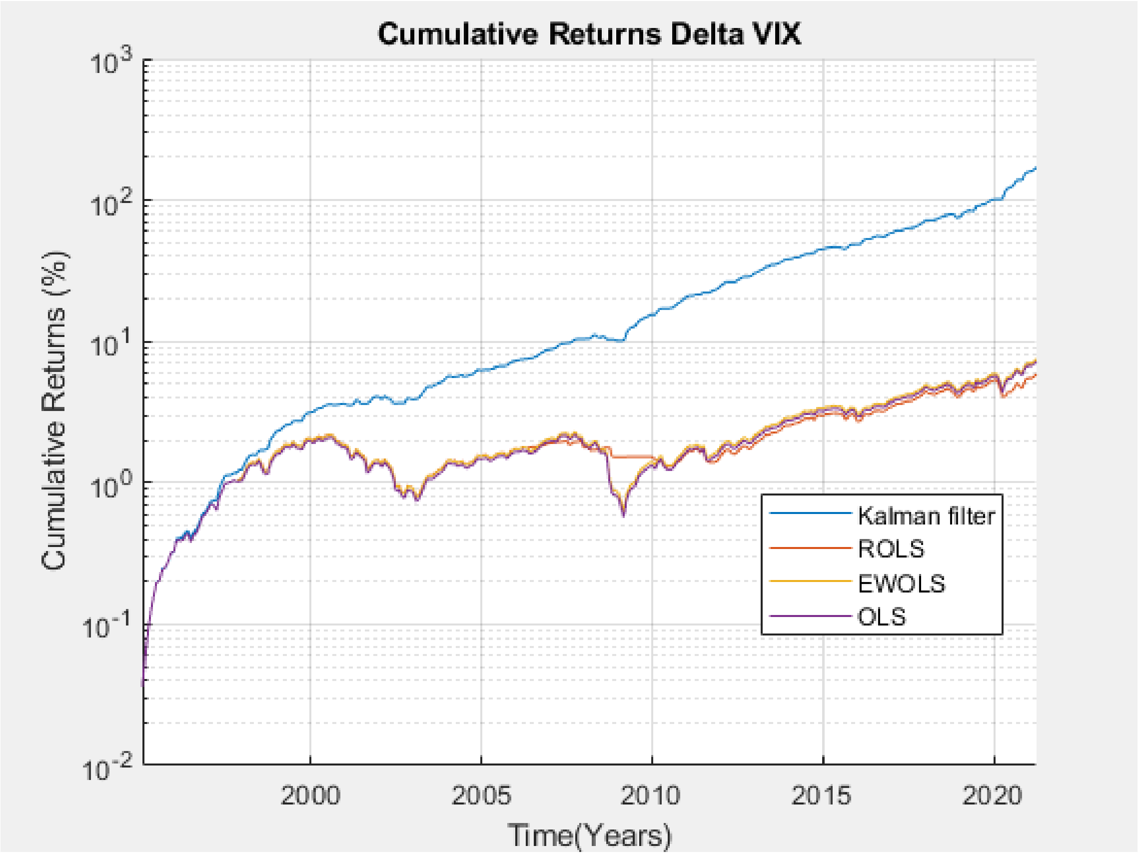
Market timing strategy performance summary.

This table reports the market timing strategy results for each modeling technique for period January 31, 1995, to March 31, 2021. Maximum drawdown (MaxDD) is defined as the maximum observed loss (peak to trough) the investment experiences during the 26.25 year period. Maximum drawdown months (MaxDDM) is the maximum number of months the investment has seen between peaks. This table additionally restates each modeling technique's goodness-of-fit measures.

					Strategy performance					
Modeling Technique	Variable	Anlzd. TWR	Vol. σ	Sharpe Ratio	Anlzd. SR	Max DD	Max DDM	RMSE	MAE	$OOSR^2$
KF	ΔVIX	21.64%	2.47%	0.60	2.08	9.46%	3	3.76%	2.73%	2.62%
	Lag S&P	10.20%	3.13%	0.22	0.76	22.62%	38	3.78%	2.72%	1.65%
	D/P	8.58%	3.67%	0.16	0.55	11.41%	17	3.79%	2.77%	0.87%
EWOLS	ΔVIX	8.50%	3.81%	0.15	0.52	25.64%	17	3.82%	2.78%	-0.43%
	Lag S&P	10.13%	2.71%	0.24	0.83	23.34%	18	3.77%	2.72%	2.35%
	D/P	8.50%	3.81%	0.15	0.52	47.30%	17	3.82%	2.78%	-0.41%
ROLS	ΔVIX	7.63%	3.26%	0.15	0.52	24.94%	30	3.84%	2.83%	-1.17%
	Lag S&P	8.51%	3.12%	0.18	0.62	31.38%	30	3.79%	2.77%	1.50%
	D/P	8.14%	3.40%	0.16	0.55	34.59%	52	3.79%	2.77%	1.50%
OLS	ΔVIX	8.35%	3.81%	0.15	0.52	42.22%	17	3.80%	2.76%	0.54%
	Lag S&P	10.37%	2.63%	0.26	0.90	20.44%	8	3.77%	2.71%	2.12%
	D/P	8.47%	3.81%	0.15	0.52	46.55%	17	3.81%	2.76%	0.42%
Buy & Hold Strategy		7.94%	3.81%	0.15	0.52	38.72%	110	-	-	-
No. obs.		315	315	315	315	315	315	315	315	315

Market Timing Strategy Performance





Pair Trading Strategy—Pair Selection

- From a universe of 676 securities on NYSE and NASDAQ that met certain requirements
  - Trading since 2018
  - Minimum tick size of .01 cents
  - Minimum close > $5.00
  - Average 20-day volume >= 1.5mil shares per day
- 228, 150 distinct pairs of securities are formed and tested for cointegration
  - Engle-Granger test
  - Most negative Augmented Dickey fuller statistic
  - Select pair from same industry
- Pair selected VISA (V) and Fidelity National Information Services (FIS)
  - Daily price data
  - In-sample January 2nd, 2020 – May 31st, 2021
  - Out-of-sample June 17, 2021 – November 16, 2021

Pairs Trading Strategy

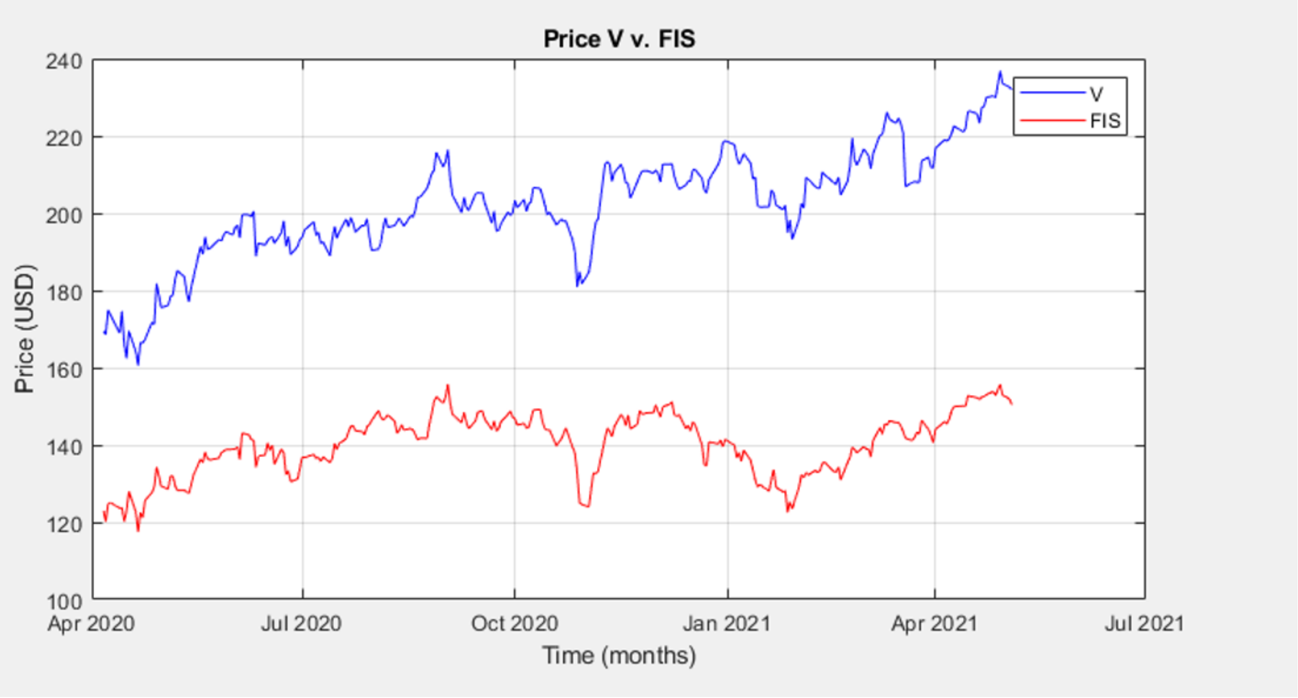
- Cointegrated securities not only move in the same direction but evolve around a shared mean
- Pairs trading exploits this relationship by monitoring the pair for temporary deviations from the mean
- An arbitrage opportunity exists when either series sufficiently departs from equilibrium

Kalman Filter

$$\text{VISA}_t = \beta_{t-1}\text{FIS}_{t-1} + \varepsilon_t \quad \sim N(0, R)$$

$$\beta_t = \beta_{t-1} + v_t \quad \sim N(0, Q)$$

- VISA (V) and Fidelity National Information Services, Inc (FIS)



Signal Generation

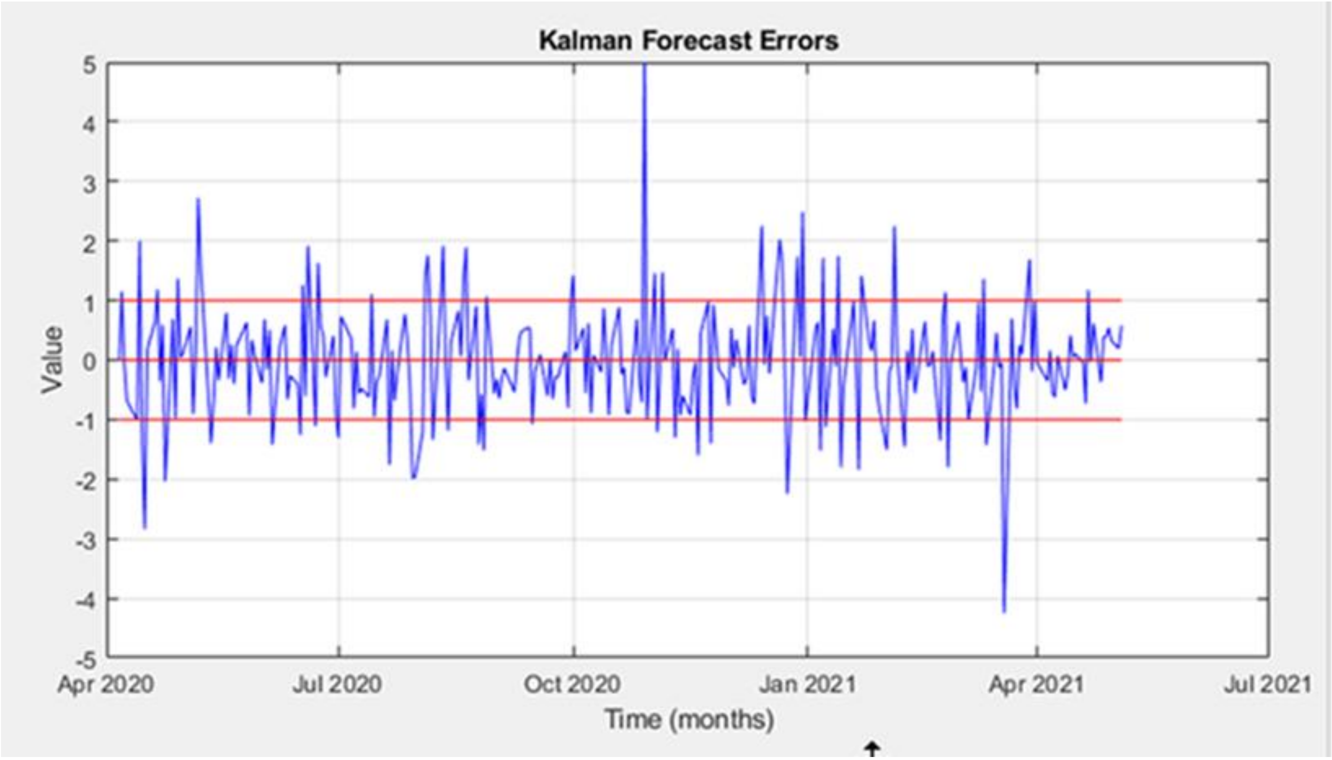
- The standardized forecast errors are monitored for substantial deviations from the mean

$$\varepsilon_t = \text{VISA}_t - (\beta_{t-1}\text{FIS}_{t-1})$$

- If the error exceeds ± 1 sd. a trade signal is generated

- If the deviation is sufficiently large and positive, short the pair—which means short the base security and go long the hedge security.

- If sufficiently large and negative, long the pair—go long the base security and short the hedge security



Strategy Results

Table 4
Model development sample strategy performance.
Development sample performance using MATLAB for January 2nd, 2020, to May 31st, 2021.

Measure	*Value*
Annualized Total Return	31%
Annualized Sharpe Ratio	2.50
Percent Winning Trades	57%
Percent Losing Trades	43%
Maximum Draw Down	4.97%
Maximum Draw Down Days	63
Number of Observations	515

Table 5
Simulated live trading results.
Performance results for strategy implemented in Interactive Brokers for 107 trading days, the period June 17th, 2021, to November 16th, 2021.

Measure	*Value*
Annualized Total Return	21%
Annualized Sharpe Ratio	1.51
Percent Winning Trades	56%
Percent Losing Trades	44%
Maximum Draw Down	3.5%
Maximum Draw Down Days	25
Number of Observations	107

Simulated Live Trading Results

Table 6

Net Asset Value (NAV) summary.

This table reports the starting and ending net asset value for the period June 17th, 2021, to November 16th, 2021 (107 trading days).

CHANGE IN NAV	**TOTAL**	
Starting Value	49,425.32	
Mark-to-Market	4,664.96	
Dividends	0.86	
Interest	-2.92	
Commissions	-154.62	
Ending Value	53,933.60	
Annualized Total Return		20.55%

Conclusions

- We have evaluated alternative time-varying coefficient forecast modeling techniques
- Measures of forecast error and out-of-sample R^2 overwhelmingly support the Kalman Filter approach
- Relative to an investor using the buy & hold strategy, an investor using time-varying coefficient models to time the market could have earned consistently positive gains—Kalman filter highest annualized total return of **21.64%**
- Deployment of the Kalman filter in a simulated live pairs trading environment produced significant economic returns—annualized total return of **20.55%**
- Future research may explore using intraday trading and alternative data sampling with the Kalman filter, i.e., volume, tick, or dollar bars instead of time bars (López de Prado, 2018)

Thank you for your time!

Market Timing Strategy Performance